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# A model for thermocouple sensitivity during microwave heating

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**Abstract**—A mathematical model is proposed to understand the sensitivity of internal temperature measurements recorded by thermocouples during microwave heating experiments. In situations of internal heating, such as when microwaves are used, the portion of the metal thermocouple guard protruding from the material body remains relatively cool and represents an effective path for thermal loss. This suggests that the temperature recorded at the tip of the embedded portion of the thermocouple may be strongly dependent on the ability of the metal guard to transfer heat away from this tip. The analytical results derived here support this thesis. Copyright © 1996 Elsevier Science Ltd.

## 1. INTRODUCTION

The mathematical problem to be investigated below is proposed as a model to assist in understanding some temperature anomalies which have been noted during microwave heating experiments (see for example [1]). These anomalies have fostered the claim that the microwave heating of certain materials yields an enhanced effect, wherein chemical changes occur at ostensibly lower temperatures than is recorded during kiln heating. The nature of this enhancement has been questioned in ref. [2]. It is suggested that the temperature discrepancies might simply be a misinterpretation of the thermocouple measurements recorded during microwave heating. The results derived here from the theoretical analysis of our proposed model support the experimental evidence of ref. [2].

The goal of ref. [2] was to demonstrate experimentally that internal temperature measurements are very sensitive to the thermocouple configuration used during the microwave heating of certain ceramic specimens. The proposed thesis is that the metal guard of the embedded thermocouple, which is needed as a shield from the microwaves, provides a very effective conductor of heat away from the internally heated specimen. In a microwave oven, that portion of the thermocouple which protrudes from the specimen remains relatively cool as compared with the uniform heating achieved in a kiln. The results of ref. [2] show that the recorded temperatures for cement mortar were dramatically decreased when the thickness of the metal guard was increased.

Here we consider a two-dimensional model as a simplified representation of a thermocouple embedded in a slab of internally heated material. While this model is only a caricature of the actual physical system, it will serve to demonstrate the sensitive relationship between the end-temperature and the thickness of a high conductivity guard embedded in a hot slab of relatively low conductivity material.

## 2. MATHEMATICAL MODEL

Figure 1 provides a sketch of the thermocouple guard inserted into a body of heated material. The metal guard is shown as a closed end tube partially embedded in the internally heated material. The thermocouple, not shown, is in physical contact with the tip of the guard, and extends upward to a shielded recording apparatus. It is assumed that the thermocouple itself does not provide significant thermal loss. Figure 2 depicts the relevant portion of the full sketch which has been geometrically simplified for purposes of the mathematical model. In Fig. 2, both the thermocouple guard and the body are represented as rectangular strips which are joined along a common interface representing the embedded portion of the thermocouple.

Referring to the two-dimensional model of Fig. 2, we consider a steady state condition in which the

**NOMENCLATURE**

| | |
|---|---|
| $A, B$ | parameter groups |
| $g$ | Green's function for thermocouple guard domain |
| $G$ | Green's function for body domain |
| $h$ | thermocouple guard penetration depth |
| $k_1$ | thermocouple guard thermal conductivity |
| $k_2$ | body thermal conductivity |
| $l$ | thermocouple guard wall thickness |
| $L$ | body dimension |
| $N_n$ | normalization constants |
| $q$ | nondimensional heating parameter |
| $Q$ | microwave heating parameter |
| $T_0$ | ambient temperature |
| $T_1$ | thermocouple guard temperature |
| $T_2$ | body temperature |
| $u$ | nondimensional thermocouple guard temperature |
| $U$ | nondimensional body temperature |
| $x, y$ | nondimensional coordinates for thermocouple guard domain |
| $X, Y$ | rectangular coordinates. |

Greek symbols

| | |
|---|---|
| $\hat{\alpha}_1$ | convection coefficient for thermocouple guard |
| $\hat{\alpha}_2$ | convection coefficient for body |
| $\alpha_1, \alpha_2$ | parameter groups |
| $\beta$ | parameter group |
| $\lambda_n$ | eigenvalues |
| $\mu$ | parameter group |
| $\xi, \eta$ | nondimensional coordinates for body domain. |

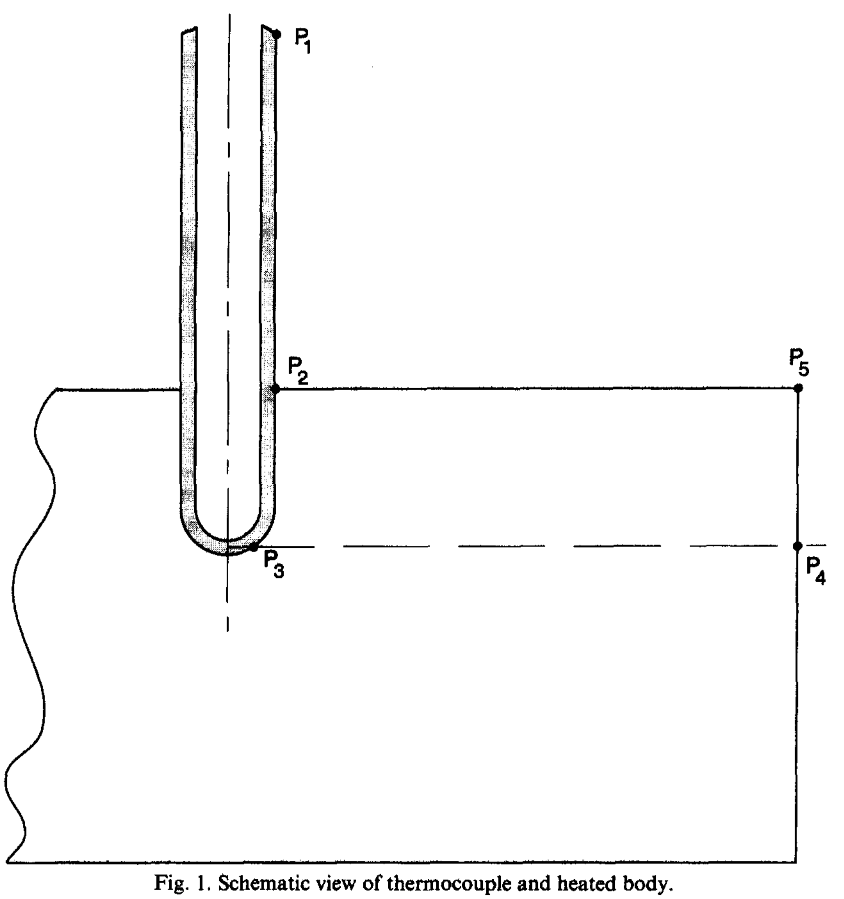


Fig. 1. Schematic view of thermocouple and heated body.

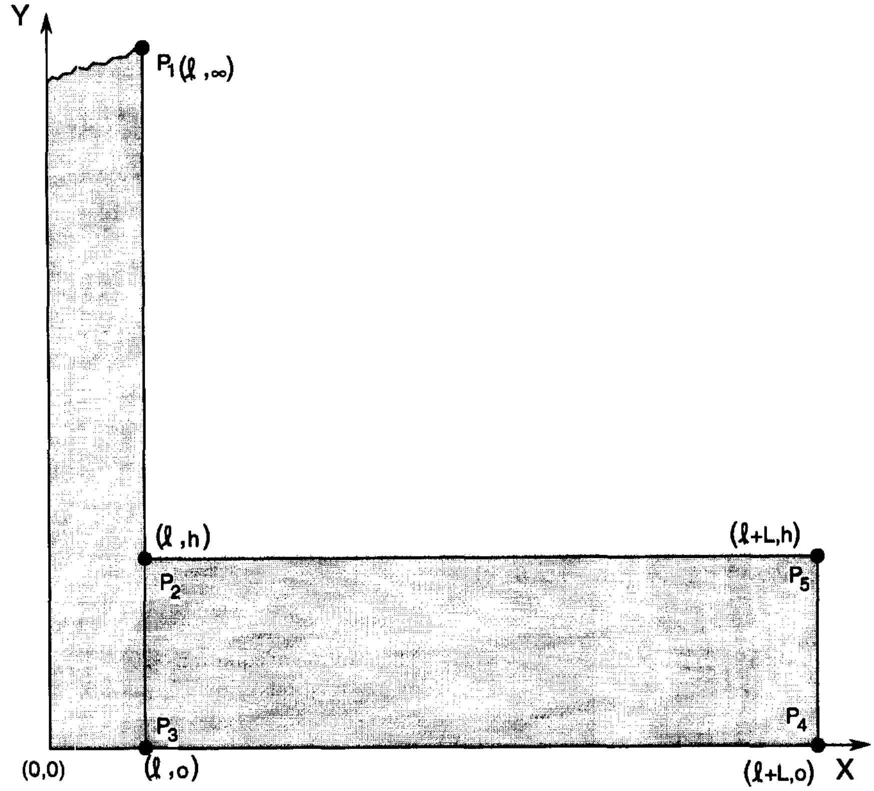

Fig. 2. Simplified geometry of thermocouple and heated body.

temperature $T_1$ in the thermocouple guard of thickness $l$ satisfies

$$\frac{\partial^2 T_1}{\partial X^2}(X, Y) + \frac{\partial^2 T_1}{\partial Y^2}(X, Y) = 0,$$

$$0 < X < l, \quad Y > 0 \tag{1}$$

$$\frac{\partial T_1}{\partial X}(0, Y) = 0, \quad Y \geqslant 0 \tag{2}$$

$$k_1 \frac{\partial T_1}{\partial X}(l, Y) = -\hat{\alpha}_1 [T_1(l, Y) - T_0], \quad Y > h \tag{3}$$

$$\frac{\partial T_1}{\partial Y}(X, 0) = 0, \quad T_1(X, \infty) = T_0, \quad 0 \leqslant X \leqslant l. \tag{4}$$

The temperature $T_2$ in the microwave heated body with a characteristic dimension $L$ is required to satisfy

$$k_2 \left[ \frac{\partial^2 T_2}{\partial X^2}(X, Y) + \frac{\partial^2 T_2}{\partial Y^2}(X, Y) \right]$$

$$= -Q, \quad l < X < l+L, \quad 0 < Y < h \tag{5}$$

$$k_2 \frac{\partial T_2}{\partial X}(l+L, Y)$$

$$= -\hat{\alpha}_2 [T_2(l+L, Y) - T_0], \quad 0 \leqslant Y \leqslant h \tag{6}$$

$$\frac{\partial T_2}{\partial Y}(X, 0) = 0, \quad \frac{\partial T_2}{\partial Y}(X, h) = 0, \quad l \leqslant X \leqslant l+L. \tag{7}$$

Along the common interface, the continuity of temperature and thermal flux require that

$$T_1(l, Y) = T_2(l, Y),$$

$$k_1 \frac{\partial T_1}{\partial X}(l, Y) = k_2 \frac{\partial T_2}{\partial X}(l, Y), \quad 0 \leqslant Y \leqslant h. \tag{8}$$

In this dimensional form of the model, the thermal conductivities for the thermocouple and body are $k_1$ and $k_2$, respectively, while the convection coefficients are $\alpha_1$ and $\alpha_2$, respectively. The internal heating by the microwaves is taken to be a constant $Q$. The relatively cool ambient temperature within the microwave oven is denoted by $T_0$.

The boundary conditions (2)–(4) and (6)–(7) reflect some simplifying assumptions implicit in the model. It is assumed that no heat is lost from the metal guard to the interior of the thermocouple. Also, no heat from the body enters the small tip of the guard at $Y = 0$. It is further specified that the body is insulated along its exposed surface at $Y = h$. Moreover, it is assumed that any vertical heat transfer within the body below $Y = 0$ can be neglected so as to justify a no flux condition along this surface. Convection loss is allowed (i) along the exposed portion of the metal guard, $X = l$, $Y > h$; (ii) at the non-insulated surface of the body, $X = l+h$, $0 \leqslant Y \leqslant h$. In essence, the model emphasizes the horizontal conduction of heat in the body to either its exterior surface at $X = l+L$, where it can be convected away, or to its interface with the metal guard at $X = l$, where it is then conducted vertically upward along the guard toward its remote end.

To put (1)–(8) into a more suitable form for analysis, we introduce the nondimensional variables

$$x = X/l, \quad y = Y/l,$$
$$u(x,y) = [T_1(X,Y)-T_0]/T_0 \quad \xi = (X-l)/L,$$
$$\eta = Y/L, \quad U(\xi,\eta) = [T_2(X,Y)-T_0]/T_0 \qquad (9)$$

and parameters

$$\alpha_1 = \hat{\alpha}_1 l/h, \quad \alpha_2 = \hat{\alpha}_2 L/k_2,$$
$$\beta = k_2 l/k_1 L, \quad q = QL^2/k_1 T_0. \qquad (10)$$

Then equations (1)–(8) become

$$\frac{\partial^2 u}{\partial x^2}(x,y)+\frac{\partial^2 u}{\partial y^2}(x,y) = 0, \quad 0 < x < 1, \quad y > 0 \qquad (11)$$

$$\frac{\partial u}{\partial x}(0,y) = 0, \quad y \geqslant 0 \qquad (12)$$

$$\frac{\partial u}{\partial x}(1,y) = -\alpha_1 u(1,y), \quad y > h/l \qquad (13)$$

$$\frac{\partial u}{\partial y}(x,0) = 0, \quad u(x,\infty) = 0, \quad 0 \leqslant x \leqslant 1 \qquad (14)$$

$$\frac{\partial^2 U}{\partial \xi^2}(\xi,\eta)+\frac{\partial^2 U}{\partial \eta^2}(\xi,\eta) = -q, \quad 0 < \xi < 1, \quad 0 < \eta < h/L \qquad (15)$$

$$\frac{\partial U}{\partial \xi}(1,\eta) = -\alpha_2 U(1,\eta), \quad 0 \leqslant \eta \leqslant h/L \qquad (16)$$

$$\frac{\partial U}{\partial \eta}(\xi,0) = 0, \quad \frac{\partial U}{\partial \eta}(\xi,h/L) = 0, \quad 0 \leqslant \xi \leqslant 1 \qquad (17)$$

$$u(1,y) = U(0,\eta), \quad \frac{\partial u}{\partial x}(1,y) = \beta\frac{\partial U}{\partial \xi}(0,\eta), \qquad (18)$$
$$0 \leqslant y \leqslant h/l, \quad 0 \leqslant \eta \leqslant h/L.$$

Our goal is to determine information about the temperature at the tip of the thermocouple from the solution of equations (11)–(18).

### 3. SOLUTION FOR THE TIP TEMPERATURE

The overall approach to the solution of equations (11)–(18) will be to first obtain separate representations for the temperature field in the metal guard and in the internally heated body, using Green's functions. Then, the matching conditions, equation (18), at the interface will be applied. This will then lead to the determination of the temperature at the tip of the thermocouple.

An integral representation of $u(x,y)$ which satisfies equations (11)–(14) is given by

$$u(x,y) = \int_0^{h/l} g(x,y|1,y_0)\left[\frac{\partial u}{\partial x}(1,y_0)+\alpha_1 u(1,y_0)\right]\mathrm{d}y_0 \qquad (19)$$

where $g(x,y|x_0,y_0)$ is the solution of the Green's function problem

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial x^2}\right)g(x,y|x,y_0) = -\delta(x-x_0)\delta(y-y_0), \quad 0 < x < l, \quad y > 0 \qquad (20)$$

$$\frac{\partial g}{\partial x}(0,y|x_0,y_0) = 0, \quad y > 0 \qquad (21)$$

$$\frac{\partial g}{\partial x}(1,y|x_0,y_0) = -\alpha_1 g(1,y|x_0,y_0), \quad y \geqslant 0 \qquad (22)$$

$$\frac{\partial g}{\partial y}(x,0|x_0,y_0) = 0,$$
$$u(x,\infty|x_0,y_0) = 0, \quad 0 \leqslant x \leqslant 1. \qquad (23)$$

The solution of equations (20)–(23) can be determined by the methods shown in ref. [3]. The result can be expressed as

$$g(x,y|x_0,y_0) = \sum_{n=1}^{\infty}\frac{1}{N_n\sqrt{\lambda_n}}\cos(\sqrt{\lambda_n}x_0) \times \cos(\sqrt{\lambda_n}x)\cosh(\sqrt{\lambda_n}y_<)\mathrm{e}^{-\sqrt{\lambda_n}y_>}, \qquad (24)$$

where

$$y_> = \begin{cases} y, & y > y_0 \\ y_0, & y < y_0 \end{cases} \quad y_< = \begin{cases} y, & y < y_0 \\ y_0, & y > y_0 \end{cases}. \qquad (25)$$

The eigenvalues $\lambda_n$ in equation (24) are determined by the transcendental equation

$$\sqrt{\lambda_n}\tan\sqrt{\lambda_n} = \alpha_1, \quad n = 1,2,\ldots \qquad (26)$$

and the normalizing constant is given by

$$N_n = \frac{1}{2}+\frac{(\sin\sqrt{\lambda_n})^2}{2\alpha_1}. \qquad (27)$$

An integral representation of $U(\xi,\eta)$ which satisfies equations (15)–(17) is given by

$$U(\xi,\eta) = \int_0^{h/L}\int_0^1 G(\xi,\eta|\xi_0,\eta_0)q\,\mathrm{d}\xi_0\,\mathrm{d}\eta_0 + \int_0^{h/L} G(\xi,\eta|0,\eta_0)\frac{\partial U}{\partial \xi}(0,\eta_0)\,\mathrm{d}\eta_0 \qquad (28)$$

where $G(\xi,\eta|\xi_0,\eta_0)$ is the solution of the Green's function problem,

$$\left(\frac{\partial^2}{\partial \xi^2}+\frac{\partial^2}{\partial \eta^2}\right)G(\xi,\eta|\xi_0,\eta_0) = -\delta(\xi-\xi_0)\delta(\eta-\eta_0),$$
$$0 < \xi < 1, \quad 0 < \eta < \frac{h}{L} \qquad (29)$$

$$\frac{\partial G}{\partial \xi}(0,\eta|\xi_0,\eta_0) = 0,$$

$$\frac{\partial G}{\partial \xi}(1,\eta|\xi_0\eta_0) = -\alpha_2 G(1,\eta|\xi_0,\eta_0), \quad (30)$$

$$0 \leqslant \eta \leqslant \frac{h}{L}$$

$$\frac{\partial G}{\partial \eta}(\xi,0|\xi_0,\eta_0) = 0,$$

$$\frac{\partial G}{\partial \eta}\left(\xi,\frac{h}{L}\middle|\xi_0,\eta_0\right) = 0, \quad 0 \leqslant \xi \leqslant 1. \quad (31)$$

As before, the solution of equations (29)–(31) can be determined by the methods shown in ref. [3]. The result can be expressed as

$$G(\xi,\eta|\xi_0,\eta_0) = \sum_{n=0}^{\infty} G_n(\xi|\xi_0)\cos\left(\frac{n\pi L\eta_0}{h}\right)\cos\left(\frac{n\pi L\eta}{h}\right) \quad (32)$$

where

$$G_0(\xi|\xi_0) = \frac{L}{\alpha_2 h}[1+\alpha_2(1-\xi_>)]$$

$$G_n(\xi,\xi_0) = \frac{2}{n\pi}\cosh\left(\frac{n\pi L\xi_<}{h}\right) \times \left\{D_n\cosh\left(\frac{n\pi L\xi_>}{h}\right) - \sinh\left(\frac{n\pi L\xi_>}{h}\right)\right\}$$

$$D_n = \frac{n\pi L\cosh\left(\frac{n\pi L}{h}\right)+h\alpha_2\sinh\left(\frac{n\pi L}{h}\right)}{n\pi L\sinh\left(\frac{n\pi L}{h}\right)+h\alpha_2\cosh\left(\frac{n\pi L}{h}\right)},$$

$$n = 1,2,\ldots. \quad (33)$$

Here $\xi_>$ and $\xi_<$ are defined analogously to equation (25).

The expressions for $u(x,y)$ and $U(\xi,\eta)$ given by equations (19) and (28), respectively, are not fully determined because of the unknown quantities which appear in the integrands. The determination of these unknown quantities is achieved by imposing equation (18). It follows from equations (18) and (19) that

$$u(x,y) = \int_0^{h/l} g(x,y|1,y_0) \times\left[\beta\frac{\partial U}{\partial \xi}\left(0,\frac{ly_0}{L}\right)+\alpha_1 U\left(0,\frac{ly_0}{L}\right)\right]\mathrm{d}y_0. \quad (34)$$

In equation (28), the fact that $q$ is constant leads to

$$U(\xi,\eta) = q\left[\frac{1}{\alpha_2}+\frac{1}{2}(1-\xi^2)\right] - \int_0^{h/L} G(\xi,\eta_0)\frac{\partial U}{\partial \xi}(0,\eta_0)\,\mathrm{d}\eta_0. \quad (35)$$

Since our interest is only in the temperature at the tip of the interface between the body and the thermocouple, we will confine our analysis of equations (34) and (35) to the determination of $U(0,0)$. To facilitate the finding of $U(0,0)$, we utilize that the geometry of the model implies that

$$\frac{l}{L} \ll 1, \quad \frac{h}{L} \ll 1, \quad (36)$$

and retain only the leading order contributions from the integrals in equations (34) and (35). It follows from equations (18) and (34) that

$$\begin{aligned} U(0,0) = u(1,0) &= \int_0^{h/L} g(1,0|1,y_0) \times\left[\beta\frac{\partial U}{\partial \xi}\left(0,\frac{ly_0}{L}\right)+\alpha_1 U\left(0,\frac{ly_0}{L}\right)\right]\mathrm{d}y_0 \\ &\approx \left[\beta\frac{\partial U}{\partial \xi}(0,0)+\alpha_1 Y(0,0)\right] \times \int_0^{h/L} g(1,0|1,y_0)\,\mathrm{d}y_0 \\ &= \left[\beta\frac{\partial U}{\partial \xi}(0,0)+\alpha_1 U(0,0)\right] \times \sum_{n=1}^{\infty}\frac{\cos^2\sqrt{\lambda_n}}{\lambda_n N_n}\left(1-\mathrm{e}^{-(\sqrt{\lambda_n}h/l)}\right) \end{aligned} \quad (37)$$

while equations (35) yields

$$\begin{aligned} U(0,0) &= \frac{q}{\alpha_2}\left(1+\frac{\alpha_2}{2}\right) - \int_0^{h/L} G(0,0|0,\eta_0)\frac{\partial U}{\partial \xi}(0,\eta_0)\,\mathrm{d}\eta_0 \\ &\approx \frac{q}{\alpha_2}\left(1+\frac{\alpha_2}{2}\right) - \left[\frac{\partial U}{\partial \xi}(0,0)\right] \times \int_0^{h/L} G(0,0|0,\eta_0)\,\mathrm{d}\eta_0 \\ &= \frac{q}{\alpha_2}\left(1+\frac{\alpha_2}{2}\right) - \left[\frac{\partial U}{\partial \xi}(0,0)\right]\left(1+\frac{1}{\alpha_2}\right). \end{aligned} \quad (38)$$

Solving equations (37)–(38) for $U(0,0)$ by elimination of $\partial U/\partial\xi(0,0)$ gives

$$U(0,0) = \frac{\beta q\left(1+\frac{\alpha_2}{2}\right)\sum_{n=1}^{\infty}\frac{\cos^2\sqrt{\lambda_n}}{\lambda_n N_n}\left(1-e^{-(\sqrt{\lambda_n}h/l)}\right)}{1+\alpha_2+[\beta\alpha_2-\alpha_1(1+\alpha_2)]\sum_{n=1}^{\infty}\frac{\cos^2\sqrt{\lambda_n}}{\lambda_n N_n}\left(1-e^{-(\sqrt{\lambda_n}h/l)}\right)}. \tag{39}$$

Thus, equation (39) provides the approximate behavior of temperature at the tip of the thermocouple, under the assumptions of equation (36). Additional simplification of the form of equation (39) results from the very realistic assumption that the thermocouple guard is sufficiently thin so that

$$\alpha_1 = \frac{\hat{\alpha}_1 l}{k_1} \ll 1. \tag{40}$$

Under this additional assumption, it follows from equations (26) and (27) that

$$\lambda_1 \approx \alpha_1, \quad \lambda_n \approx (n-1)\pi + \frac{\alpha_1}{(n-1)\pi}, \quad n = 2,3,\ldots,$$

$$N_1 \approx 1, \quad N_n \approx \frac{1}{2} + \left[\frac{\alpha_1}{(n-1)\pi}\right]^2, \quad n = 2,3,\ldots. \tag{41}$$

This implies that the series in equation (39) can be rather well approximated by its leading term when equation (40) holds. Then equation (39) can be replaced by

$$U(0,0) \approx \frac{\beta q\left(1+\frac{\alpha_2}{2}\right)\left(1-e^{-(\sqrt{\alpha_1}h/l)}\right)}{\alpha_1(1+\alpha_2)+[\beta\alpha_2-\alpha_1(1+\alpha_2)]\left(1-e^{-(\sqrt{\alpha_1}h/l)}\right)}. \tag{42}$$

## 4. TIP TEMPERATURE BEHAVIOR

Let us recall that the motivation for deriving an expression for $U(0,0)$ was to determine how sensitively the temperature at the tip of the thermocouple depends upon the thickness $l$ of the metal guard. To examine this relationship, we convert equation (42) back into an equivalent dimensional form through equations (9) and (10). It follows that

$$T_2(l,0) = T_0 + \frac{A\left(1+\frac{B}{2}\right)\left(1-e^{-(\mu/\sqrt{l})}\right)}{(1+B)+\left(\frac{\hat{\alpha}_2}{\hat{\alpha}_1}-1-B\right)\left(1-e^{-(\mu/\sqrt{l})}\right)} \tag{43}$$

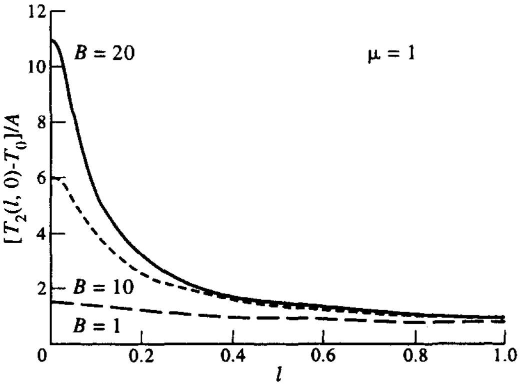


Fig. 3. Tip temperature dependence on guard thickness [equation (47)].

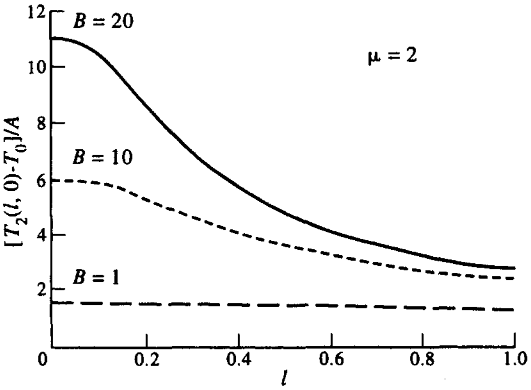


Fig. 4. Tip temperature dependence on guard thickness [equation (47)].

where

$$A = \frac{k_2 Q L}{\hat{\alpha}_1 k_1}, \quad B = \frac{\hat{\alpha}_2 L}{k_2}, \quad \mu = h\sqrt{\frac{\hat{\alpha}_1}{k_1}}. \tag{44}$$

This expression, equation (43), for the dimensional tip temperature $T(l,0)$ indicates the explicit dependence on the thickness $l$. Differentiation of equation (43) with respect to $l$ yields

$$\frac{dT_2}{dl}(l,0) = -\frac{A(1+B)\left(1+\frac{B}{2}\right)\mu e^{-(\mu/\sqrt{l})}}{2l^{3/2}\left[(1+B)+\left(\frac{\hat{\alpha}_2}{\hat{\alpha}_1}-1-B\right)\left(1-e^{-(\mu/\sqrt{l})}\right)\right]^2}. \tag{45}$$

Since this derivative is negative, then $T_2(l,0)$ is a monotonically decreasing function of $l$. Moreover, the derivative vanishes at the extreme values, $l = 0$ and $l \to \infty$, so that $T_2(l,0)$ will be most sensitive to changes in $l$ at some intermediate value $l^*$. That value of maximum sensitivity corresponds to the inflection point found by differentiation of equation (45). It follows that $l^*$ is determined by the transcendental equation

$$6\frac{\hat{\alpha}_2}{\hat{\alpha}_1}\sqrt{l^*}(\mu+3\sqrt{l^*})^{-1} = \frac{\hat{\alpha}_2}{\hat{\alpha}_1}-\left(\frac{\hat{\alpha}_2}{\hat{\alpha}_1}-1-B\right)\exp\left(-\frac{\mu}{\sqrt{l^*}}\right). \quad (46)$$

A special case of some interest is that in which the convection coefficients for the body and the thermocouple can be regarded as equal. If $\hat{\alpha}=\hat{\alpha}_1=\hat{\alpha}_2$, then equation (43) becomes

$$T_2(l,0) = T_0+\frac{A\left(1+\frac{B}{2}\right)\left(1-e^{-(\mu/\sqrt{l})}\right)}{1+Be^{-(\mu/\sqrt{l})}}. \quad (47)$$

To illustrate the potential sensitivity of the tip temperature to the thickness of the metal guard, we have graphically displayed the behavior of equation (47) in Figs. 3 and 4 for a selection of values of the parameters $\mu$ and $B$. It is seen that greater sensitivity is associated with larger values of $B$, with the most sensitive guard thickness $l^*$ being relatively small.

Our conclusion from the analysis of the mathematical model presented here is that the tip temperature of the thermocouple can be extremely sensitive to the thickness of a thin metal guard during microwave heating. These results imply a qualitative confirmation of the thesis proposed in ref. [2]. Further experimental studies are being planned in which some quantitative comparison can be made with the theory developed here.

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